TWO PERPENDICULARS TO A TRANSVERSAL.

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Do two perpendiculars to a transversal intersect ?

Both Euclid and Lobatschewsky affirm that they do not. Euclid regards the two perpendiculars as equidistant, whilst Lobatschewsky considers them as diverging.

Experience confirms the view that the distance between the perpendiculars is a constant. As long as this is the case it is evident that intersection is impossible. If the perpendiculars do not approach each other within the range of observation and experience what would anology and induction indicate ? Would they not unmistakably favor the hypothesis that the perpendiculars do not intersect beyond the limits of observation and experience ? Our knowledge of *the here* and *the now*, if at all accurate, must assuredly count for something *elsewhere* and *tomorrow*.

But aside from conclusions based on purely empirical data and obtained by analogical and inductive processes the assumption that a straight line that has a beginning and an end is *infinite* involves contradiction and is therefore absurd. One end of each perpendicular is at the transversal. If these perpendiculars intersect each of them has two ends. But *two* ends is the distinctive characteristic of a finite straight line.

The further assumption that the intersection takes place at a hypothetical place called "infinity" does not remove the difficulty in the slightest. *Two* ends are still attributed to the *supposed* infinite line.

There is in reality a new difficulty and a very serious one, for the logical law of non-contradiction is violated.

The difficulty is not that the human mind by reason of its limited powers is unable to cognize an unlimited straight line and discover what will or will not take place "at infinity," but it is that the mind by reason of the logical law of non-contradiction can not cognize a line that is at the same time both unlimited and limited.

As a result of this brief investigation we find that there are insuperable difficulties, logical, geometrical, and philosophical, in the hypothesis that two perpendiculars to a transversal intersect at a supposed place called "infinity."

Notwithstanding these difficulties in the way of this hypothesis many analysts daily and habitually accept it. They do make the "assumption that parallel lines, extended to an infinite distance, do intersect."

Euclid flatly contradicts this hypothesis in his statement that "parallels never meet however far they may be produced." In favor of Euclid's statement there is nothing in logic, science or geometry known to man that conflicts with it. I understand Mr. Drummond's protest to extend not only to Euclid's assumption but also to the assumption that Euclid contradicts.

If the analysts "can not comprehend the infinite" why do they employ the symbol of the infinite so freely in their equations and decide without hesitation so many questions against the Alexandrian geometer? The analysts make large use of the symbol ∞ in their equations. Do they or do they not comprehend the meaning of the symbolism employed? If they find ∞ incomprehensible, can they not obtain all legitimate results by the aid of *finite* quantities alone?

DEVELOPMENT OF SINθ AND COSθ.

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In discussing the power of the calculus with my own students in Trinity College, I, several years ago, sprung the question "why can the trigonometric functions, sine and cosine, be developed by series?"

The calculus very readily furnished the series; but it did not expose the exponential nature of the functions.

The fact that the value of the functions can be expressed by series forced me to the conclusion that the reason existed in the nature of the functions themselves, and, therefore, they should yield this result directly.

Before proceeding to obtain the series directly from the functions, it will be necessary to produce a series involving an exponential function. The object thereafter will be to trace the law which connects sine and cosine with this exponential function.

We will develop $\left(1+\frac{1}{x}\right)^x\Big]_\infty$ which gives us a simple converging series. This series can be made to express an exponential function.

Denoting $\left(1+\frac{1}{x}\right)^x\Big]_\infty$ by e; that is, as x increases indefinitely, the *limiting value* of this function $\left(1+\frac{1}{x}\right)^x\Big]_\infty$ is e.

$\therefore\ e=1+1+\frac{1}{1.2}+\frac{1}{1.2.3}$, etc.* From this we get

$$e^\theta=\left\{\left(1+\frac{1}{x}\right)^x\Big]_\infty\right\}^\theta=1+\theta+\frac{\theta^2}{1.2}+\frac{\theta^2}{1.2.3}+\text{etc.},\ \ldots\ldots\ldots\ldots\ldots(1),$$

$$e^{\frac{1}{x}}=\left\{\left(1+\frac{1}{x}\right)^x\Big]_\infty\right\}^{\frac{1}{x}}=1+\frac{1}{\infty},\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2),$$

*This gives $e=2.71828$, the Naperian base.